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Fuzzy Maximal, Minimal A-Open And A-Closed Sets

M. Sankari

Department of Mathematics Lekshmipuram College of Arts and Science Neyyoor, Kanyakumari Tamil Nadu-629 802, India.

† Corresponding author: sankarisaravanan1969@gmail.com

Abstract
In this article we find few conditions for fuzzy α -disconnectedness using fuzzy maximal and fuzzy minimal α -open sets. We obtain some identical results by fuzzy minimal and maximal α -closed sets in connection with how these related with others. It is shown that if a fuzzy space has a set which is fuzzy minimal and fuzzy maximal, then either this fuzzy set is the only nontrivial fuzzy α -open set in the fuzzy space or the fuzzy space is fuzzy α -disconnected.

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1 Introduction and Preliminaries

After the introduction of fuzzy sets by the Zadeh[5], the concept of fuzzy topology introduced by Chang[3] in 1968. The notion of fuzzy minimal open[4] set explored by Swaminathan and Sivaraja.

In this paper, (X, τ) or X stands for fuzzy topologcal space. The symbols λ , μ , γ, η ,... are used to denote fuzzy sets and all other symbols have their usual meaning unless explicitely stated.

In this papers we define fuzzy minimal α -open and fuzzy maximal α -open sets. Moreover we study some properties and interrelations of fuzzy minimal and fuzzy maximal α -open and α -closed sets.

Definition 1.1. A fuzzy set μ in a fts is called a fuzzy α -open [2] if $\mu \leq int\ [cl(int\ \mu)]$ and a fuzzy closed if cl [int(cl μ)] .

Definition 1.2. A fuzzy topological space X is said to be fuzzy α -connected [1] if it has no proper fuzzy α -clopen set.
A fuzzy topological space X is said to be fuzzy α -disconnected if it is not fuzzy α -connected.

2 Fuzzy Maximal and Minimal α-Open Sets

Now we define fuzzy minimal α -open and fuzzy maximal α -open sets.

Definition 2.1. A proper nonzero fuzzy α -open set α of X is said to be a fuzzy minimal α -open set if α and 0_X are only fuzzy α -open sets contained in α .

Definition 2.2. A proper nonzero fuzzy α-open set α of X is said to be a fuzzy maximal α -open set if 1_X and α are only fuzzy α -open sets containing α .

Theorem 2.1. If μ is a fuzzy maximal α -open set and λ be a fuzzy α -open subset, then either $\mu \vee \lambda = 1_X$ or $\lambda < \mu$.

Theorem 2.2. If μ is a fuzzy minimal α -open set and λ be a fuzzy α -open subset, then either $\mu \wedge \lambda = 0_X$ or $\mu < \lambda$.

Theorem 2.3. If ϑ is a fuzzy maximal α -open set and α is a fuzzy minimal α -open set in a fuzzy topological space X with $\alpha < \vartheta$, then $\vartheta = 1_X - \alpha$.

Theorem 2.4. If μ is a fuzzy maximal α -open set and λ is a fuzzy minimal α -open set of a fuzzy topological space X, then either $\lambda < \mu$ or the space is fuzzy α -disconnected.

Proof. On deploying the maximality of μ by Theorem 2.1, we have either $\mu \vee \lambda = 1_X$ or $\lambda < \mu$. On deploying the minimality of λ by Theorem 2.2, we have either $\mu \wedge \lambda = 0_X$ or $\lambda < \mu$. When $\mu \vee \lambda = 1_X$, $\lambda < \mu$ gives $\mu = 1_X$; when $\mu \wedge \lambda = 0_X$, $\lambda < \mu$ gives $\lambda = 0_X$. Therefore the probable occurences are $\mu \vee \lambda = 1_X$, $\mu \wedge \lambda = 0_X$ and $\lambda < \mu$. If $\mu \vee \lambda = 1_X$, $\mu \wedge \lambda = 0_X$, then the space is fuzzy α –disconnected.

Remark 2.5. $\mu \vee \lambda = 1_X$, $\mu \wedge \lambda = 0_X$, imply $\mu = 1_X - \lambda$. In Theorem , if $\lambda < \mu$/, then μ and λ are fuzzy α -closed. Theorem may be stated as follows: if μ is a fuzzy maximal

α -open set and λ is a fuzzy minimal α -open set of a fuzzy topological space X , then either $\lambda < \mu$ or $\mu = 1_X - \lambda$.

Theorem 2.6. If a fuzzy topological space X has a fuzzy set which is both fuzzy maximal and fuzzy minimal α -open, then either this fuzzy set is the only nontrivial fuzzy α -open set in the space or the fuzzy space is fuzzy α -disconnected.

Proof. Let μ be both fuzzy maximal and fuzzy minimal α -open, and λ be any fuzzy α -open set. Then we get $\mu < \mu \vee \lambda$. By the fuzzy maximality of μ, we have the following two cases.

Case I: $\mu = \mu \vee \lambda$. Then we get $\lambda < \mu$. Since μ is fuzzy minimal, we have $\lambda = 0_X$ or $\lambda = \mu$.

Case II: $\mu \vee \lambda = 1_X$. Considering μ as a fuzzy minimal α -open set, we get by Theorem 2.2, $\mu \wedge \lambda = 0_X$ or $\mu < \lambda$. Since μ is fuzzy maximal, $\mu < \lambda$ implies $\mu = \lambda$ or $\lambda = 1_X$. Considering all the cases, we get $\mu = \lambda$ or $\mu \vee \lambda = 1_X$ and $\mu \wedge \lambda = 0_X$. If $\mu \vee \lambda = 1_X$ and $\mu \wedge \lambda = 0_X$, then the space is fuzzy α -disconnected.

It is trivial that if a fuzzy topological space X has only one proper fuzzy α -open set, then that fuzzy set is both fuzzy maximal and fuzzy minimal α -open. If there are only two proper fuzzy α -open sets in a fuzzy space and the fuzzy α -open sets are disjoint, then both are fuzzy maximal and fuzzy minimal. If μ and λ are only two proper fuzzy α -open sets in the fuzzy topological space such that $\mu < \lambda$, then μ is a fuzzy minimal α -open set and λ is a fuzzy maximal α -open set in the fuzzy space. However, there may not exist a fuzzy set which is both fuzzy maximal and fuzzy minimal α -open in a fuzzy α -disconnected space (see Example 2.9).

Corollary 2.7. If μ is both fuzzy maximal and fuzzy minimal α -open, and ϑ is a fuzzy α -closed set in a fuzzy topological space X , then either $\mu = 1_X - \vartheta$ or $\mu = \vartheta$.

Proof. Given μ is both fuzzy maximal and fuzzy minimal α -open, and ϑ is a fuzzy α -closed set. So $1_X - \vartheta$ is a fuzzy α -open set. Proceeding like the proof of Theorem 2.6, we get $\mu = 1_X - \vartheta$ or $\mu \vee (1_X - \vartheta) = 1_X$ and $\mu \wedge (1_X - \vartheta) = 0_X$. Both $\mu \vee (1_X - \vartheta) = 1_X$ and $\mu \wedge (1_X - \vartheta) = 0_X$ imply $\mu = \vartheta$.

Corollary 2.8. If μ is both fuzzy maximal and fuzzy minimal α -open in a fuzzy topological space X , then either μ is the only proper fuzzy α -open set in the fuzzy space or proper fuzzy α -open sets of the fuzzy space are μ and $1_X - \mu$ only.

Proof. Let λ be any proper fuzzy α -open set of the fuzzy space. Proceeding like the proof of Theorem 2.6, we get $\mu = \lambda$ or $\mu \vee \lambda = 1_X$ and $\mu \wedge \lambda = 0_X$. Both $\mu \vee \lambda = 1_X$ and $\mu \wedge \lambda = 0_X$ imply $\lambda = 1_X - \mu$.

Example 2.9. Let X = {a, b, c}. Then fuzzy sets $\lambda 1 = \frac{1}{a} + \frac{0}{b} + \frac{0}{c}$; $\lambda 2 = \frac{1}{a} + \frac{1}{b} + \frac{0}{c}$; $\lambda 3 = \frac{0}{a} + \frac{1}{b} + \frac{1}{c}$; $\lambda 4 = \frac{0}{a} + \frac{1}{b} + \frac{0}{c}$; are defined as follows: Consider the fuzzy topology $\tau = \{0_X, \lambda 1, \lambda 2, \lambda 3, \lambda 4, 1_X\}$. The fuzzy topolgical space (X, τ) is fuzzy α –disconnected with a separation λ1 and λ3 .

But the fuzzy space has no fuzzy α -open set which is both fuzzy maximal and fuzzy minimal α -open.

Theorem 2.10. If γ1 and γ2 are two different fuzzy maximal α -open sets of a fuzzy topological space X with γ1 ∧ γ2 is a fuzzy α -closed set, then X is fuzzy α -disconnected.

Proof. Since γ1 and γ2 are fuzzy maximal, we have $\gamma 1 \vee \gamma 2 = 1_X$. We put $\mu = \gamma 1 - \gamma 1 \wedge \gamma 2$, $\lambda = \gamma 2$ or $\mu = \gamma 1$, $\lambda = \gamma 2 - \gamma 1 \wedge \gamma 2$. We note that μ, λ are disjoint fuzzy α -open sets with $\mu \vee \lambda = 1_X$. Therefore X is fuzzy α -disconnected.

Theorem 2.11. If β is a fuzzy maximal α -open set, then either $Cl(\beta) = 1_X$ or $Cl(\beta) = \beta$.

Theorem 2.12. If there exists a fuzzy maximal α -open set which is not fuzzy dense in a fuzzy topological space, then the space is fuzzy α -disconnected.

Proof. Let β be a fuzzy maximal set which is not fuzzy dense in X . By Theorem 2.11, $\beta = Cl(\beta)$. We write $\mu = \beta$ and $\lambda = 1_X - Cl(\beta)$. Therefore (μ, λ) is a fuzzy separation for X.

3 Fuzzy Maximal and Fuzzy Minimal α-Closed Sets

Definition 3.1. A proper nonzero fuzzy α -closed set ϑ of X is said to be a fuzzy maximal α -closed set if any fuzzy α -closed set which contains ϑ is 1_X or ϑ .

Theorem 3.1. If ϑ is a fuzzy maximal α -closed set and γ is a fuzzy α -closed set, then either $\vartheta \vee \gamma = 1_X$ or $\gamma < \vartheta$.

Definition 3.2. A proper nonzero fuzzy α -closed set ϑ of X is said to be a fuzzy minimal α -closed[**?**] set if any fuzzy α -closed set which is contained in ϑ is 0_X or ϑ .

Theorem 3.2. If γ is a fuzzy minimal α -closed set and ϑ is any fuzzy α -closed set, then either $\gamma \wedge \vartheta = 0_X$ or $\gamma < \vartheta$.

Theorem 3.3. If γ is a fuzzy maximal α -closed set and ϑ is a fuzzy α -closed set, then either $\gamma \vee \vartheta = 1_X$ or $\vartheta < \gamma$. xd

Theorem 3.4. A fuzzy set ϑ in a fuzzy topological space is both fuzzy minimal and fuzzy maximal α -closed set, then either of the following is true:

(i) ϑ is the only proper fuzzy α -closed set in the fuzzy space.

(ii) If there exists another proper fuzzy α -closed set γ , then $\vartheta \vee \gamma = 1_X$ and $\vartheta \wedge \gamma = 0_X$.

Corollary 3.5. If ϑ is both fuzzy maximal and fuzzy minimal α -closed set and λ is a fuzzy α -open set in a fuzzy topological space X , then either $\vartheta = 1_X - \lambda$ or $\vartheta = \lambda$.

Corollary 3.6. If ϑ is both fuzzy maximal and fuzzy minimal α -closed in a fuzzy topological space X , then either ϑ is the only proper fuzzy α -closed set in the fuzzy space or proper fuzzy α -closed sets of the fuzzy space are ϑ and $1_X - \vartheta$ only.

It is trivial that if a fuzzy topological space X has only one proper fuzzy α -closed set, then that set is both fuzzy maximal and fuzzy minimal α -closed. If there are only two proper fuzzy α -closed sets in a fuzzy space and the fuzzy α -closed sets are disjoint, then both are fuzzy maximal and fuzzy minimal. If γ and δ are only two proper fuzzy α -closed sets in a fuzzy topological space such that $\gamma < \delta$, then γ is a fuzzy minimal α -closed set and δ is a fuzzy maximal α -closed set in the fuzzy space. However, there may not exist a fuzzy set which is both fuzzy maximal and fuzzy minimal α -closed in a fuzzy α -disconnected space . It is observed from Example 2.9 that there exist no disjoint fuzzy α -closed set in the space which is both fuzzy maximal and fuzzy maximal α -closed. From this consideration we easily conclude that there may exist fuzzy α -closed sets $\lambda 1$ and $\lambda 2$ in X such that $\lambda 1 \vee \lambda 2 = 1_X$ and $\lambda 1 \wedge \lambda 2 = 0_X$ but there may not exist a set ehich is both fuzzy maximal and fuzzy minimal α -closed.

Theorem 3.7. If μ is both fuzzy maximal α -open and fuzzy minimal α -closed, λ is a fuzzy α -closed, then either of the following is true:

(i) $\lambda < \mu < \vartheta$.

(ii) $\lambda < \mu$ and $\mu \wedge \vartheta = 0_X$.

(iii) $\mu \vee \vartheta = 1_X$ and $\mu < \vartheta$.

(iv) $\mu \vee \vartheta = 1_X$, $\mu \wedge \vartheta = 0_X$.

Proof. By Theorem 2.1, if we take μ as a fuzzy maximal α -open set, we get $\lambda < \mu$ or $\mu \vee \lambda = 1_X$. If we take, by Theorem 3.6, μ as a fuzzy minimal α -closed set, we get $\mu < \vartheta$ or $\mu \wedge \vartheta = 0_X$. $\lambda < \mu$ and $\mu < \vartheta$ imply $\lambda < \mu < \vartheta$. The remaining probable combinations are $\lambda < \mu$, $\mu \wedge \lambda = 0_X$; $\mu \vee \lambda = 1_X$, $\mu < \vartheta$ and $\mu \vee \lambda = 1_X$, $\mu \wedge \lambda = 0_X$.

Corollary 3.8. If μ is both fuzzy maximal α -open and fuzzy minimal α -closed, then μ and $1_X - \mu$ are only proper fuzzy cl α -open sets in the fuzzy space.

Proof. Let ϑ be fuzzy cl α -open in 1_X . Putting $\lambda = \vartheta$ in Theorem 3.7, we get $\mu = \vartheta$ or $\mu = 1_X - \vartheta$.

Theorem 3.9. If μ is both fuzzy maximal α -open and fuzzy maximal α -closed, ϑ is fuzzy cl α -open, then either $\vartheta < \mu$ or $\mu \vee \vartheta = 1_X$.

Proof. Similar to that of proof of Theorem 3.7 .

Theorem 3.10. If μ is both fuzzy minimal α -open and fuzzy maximal α -closed, λ is a fuzzy α -open and ϑ is fuzzy α -closed, then either of the following is true:

(i) $\vartheta < \mu < \lambda$.

(ii) $\mu < \lambda$ and $\mu \vee \vartheta = 1_X$.

(iii) $\mu \wedge \lambda = 0_X$ and $\vartheta < \mu$.

(iv) $\mu \vee \vartheta = 1_X$, $\mu \wedge \vartheta = 0_X$.

Corollary 3.11. If μ is both fuzzy minimal α -open and fuzzy maximal α -closed, then μ and $1_X - \mu$ are only proper fuzzy α -clopen sets in the space.

Theorem 3.12. Let α, μ be fuzzy α -open sets in X such that $\alpha \wedge \mu \neq 0_X, \alpha$. Then $\alpha \wedge \mu$ is a fuzzy minimal α -open set in (α, τα) if μ is a fuzzy minimal α -open set in (X, τ) .

Proof. If $\alpha \wedge \mu$ is not a fuzzy minimal α -open set in (α, τα) , there exists a fuzzy α -open set $\beta \neq 0_X$ in (α, τα) such that $\beta \leq \alpha \wedge \mu$. Since μ is a fuzzy minimal α -open set in X and $\alpha \wedge \mu \neq 0_X$ we have by Theorem 2.2, $\mu < \alpha$ which implies $\alpha \wedge \mu = \mu$. α being fuzzy α -open in X , β is also fuzzy α -open in X . Hence we get a fuzzy set β fuzzy α -open in X such that $0_X \neq \beta \leq \mu$ which is a contradiction to our assumption that μ is a fuzzy minimal α -open set in X .

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