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Mth -Neighbourly Irregular Bipolar Fuzzy Graphs

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Abstract

In this paper, mth -neighbourly irregular bipolar fuzzy graphs and mth - neighbourly totally irregular bipolar fuzzy graphs are defined. Comparative study between mth -neighbourly irregular bipolar fuzzy graph and mth -neighbourly totally irregular bipolar fuzzy graph is done. Property of mth -neighbourly irregular bipolar fuzzy graph and mth –neighbourly totally irregular bipolar fuzzy graph are discussed. mth - neighbourly irregularity on bipolar fuzzy graphs whose underlying graphs are cycle and ladder are studied.

Keywords: Bipolar fuzzy graph, degree of a vertex in bipolar fuzzy graph, irregular bipolar fuzzy graph, neighbourly irregular bipolar fuzzy graph, neighbourly totally irregular bipolar fuzzy graph.

AMS subject classification : 05C12, 03E72, 05C72.

1 Introduction

Presently, science and technology is featured with complex processes and phenomena for which complete information is not always available. For such cases, mathematical models are developed to handle various types of systems containing elements of uncertainty. A large number of these models is based on an extension of the ordinary set theory, namely, fuzzy sets. Graph theory has numerous applications to problems in computer science, electrical engineering, system analysis, operations research, economics, networking routing, and transportation. In 1965, Zadeh[13] introduced the notion of a fuzzy subset of a set as a method for representing uncertainty. Since then the theory of fuzzy sets has become a vigorous area of research in different disciplines including medical and life sciences, management sciences, social sciences, engineering, statistics, graph theory, computer networks and automata theory. In 1994, Zhang [14, 15] initiated the concept of bipolar fuzzy sets as a generalization of fuzzy sets. Bipolar fuzzy sets are an extension of fuzzy sets whose membership degree range is $[-1, 1]$. In a bipolar fuzzy set, the membership degree of an element means that the element is irrelevant to the corresponding property, the membership degree (0, 1] of an element indicates that the element some what satisfies the property, and the membership degree [−1, 0) of an element indicates that the element somewhat satisfies the implicit counter property. S. Ravi Narayanan and N.R. Santhi Maheswari [11] defined and discussed d_2 degree of a vertex in bipolar fuzzy graph. These motivates us to define m^{th} - neighbourly irregular bipolar fuzzy graph and study some of its properties.

Throughout this paper, the vertices take the membership value $A = (m^+_1, m^-_1)$ and edges take the membership value $B = (m^+_2, m^-_2)$ where $m^+_1, m^+_2 \in [0, 1]$ and $m^-_1, m^-_2 \in [-1, 0]$.

2 Preliminaries

We present some known definitions related to fuzzy graphs and bipolar fuzzy graphs for ready reference to go through the work presented in this paper.

Definition 2.1. A fuzzy graph $G : (\sigma, \mu)$ is a pair of functions (σ, μ) , where $\sigma : V \to [0,1]$ is a fuzzy subset of a non empty set V and $\mu : V X V \to [0, 1]$ is a symmetric fuzzy relation on σ such that for all u, v in V , the relation $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$ is satisfied. A fuzzy graph G is called complete fuzzy graph if the relation $\mu(u, v) = \sigma(u) \wedge \sigma(v)$ is satisfied.

Definition 2.2. A bipolar fuzzy graph with an underlying set V is defined to be the pair (A, B) , where $A = (m_1^+, m_1^-)$ is a bipolar fuzzy set on V and $B = (m_2^+, m_2^-)$ is a bipolar fuzzy set on E such that $m_2^+(x, y) \leq \min\{(m_1^+(x), m_1^+(y)\}$ and $m_2^-(x, y) \geq \max\{m_1^-(x), m_1^-(y)\}$ for all $(x, y) \in E$. Here, A is called bipolar fuzzy vertex set on V and B is called bipolar fuzzy edge set on E.

Definition 2.3. The strength of connectedness between two vertices u and v is defined as $\mu^\infty(u, v) = \sup\{\mu^k(u, v) : k = 1, 2, ...\}$, where $\mu^k(u, v) = \sup\{\mu(u, u_1) \wedge \mu(u_1, u_2) \wedge ... \wedge \mu(u_{k-1}, v) : u, u1, u2, ..., uk-1, v$ is a path connecting u and v of length $k\}$.

Definition 2.4. The positive degree of a vertex $u \in G$ is defined as $d^+(u) = \sum m_2^+(u, v)$, for $uv \in E$. The negative degree of a vertex $u \in G$ is defined as $d^-(u) = \sum m_2^-(u, v)$, for $uv \in E$ and $m_2^+(uv) = m_2^-(uv) = 0$ if uv not in E. The degree of a vertex u is defined as $d(u) = (d+(u), d-(u))$.

Definition 2.5. Let G : (A, B) be a bipolar fuzzy graph, where $A = (m_1^+, m_1^-)$ and $B = (m_2^+, m_2^-)$ be two bipolar fuzzy sets on a non empty set V . Then, G is said to be regular bipolar fuzzy graph if all the vertices of G has same degree (c_1, c_2).

Definition 2.6. Let G : (A, B) be a bipolar fuzzy graph, where $A = (m_1^+, m_1^-)$ and $B = (m_2^+, m_2^-)$ be two bipolar fuzzy sets on a non empty set V . Then, G is said to be irregular bipolar fuzzy graph if there exists a vertex which is adjacent to the vertices with distinct degrees.

Definition 2.7. Let G : (A, B) be a bipolar fuzzy graph, where $A = (m_1^+, m_1^-)$ and $B = (m_2^+, m_2^-)$ be two bipolar fuzzy sets on a non empty set V . Then, G is said to be strongly irregular bipolar fuzzy graph if all the vertices of G have distinct degree.

Definition 2.8. The total degree of a vertex $u \in V$ is denoted by td(u) and is defined as $td(u) = (td+(u), td-(u))$, where $td^+(u) = \sum m_2^+(u, v) + (m_1^+(u))$ and $td^-(u) = \sum m_2^-(u, v) + (m_1^-(u))$.

Definition 2.9. Let G : (A, B) be a bipolar fuzzy graph, where A = (m_1^+ , m_1^-) and B = (m_2^+ , m_2^-) be two bipolar fuzzy sets on a non empty set V . Then, G is said to be totally regular bipolar fuzzy graph if all the vertices of G has same total degree (c_1, c_2).

3 m^{th} -Neighbourly irregular bipolar fuzzy graph

Definition 3.1. Let G be a bipolar fuzzy graph on $G^*(V, E)$. Then G is said to be m^{th} - neighbourly irregular bipolar fuzzy graph, if every pair of vertices which are at a distance m away from each other have distinct degrees.

Example 3.2. Consider a bipolar fuzzy graph on G^* : (V, E), a cycle of length 5.

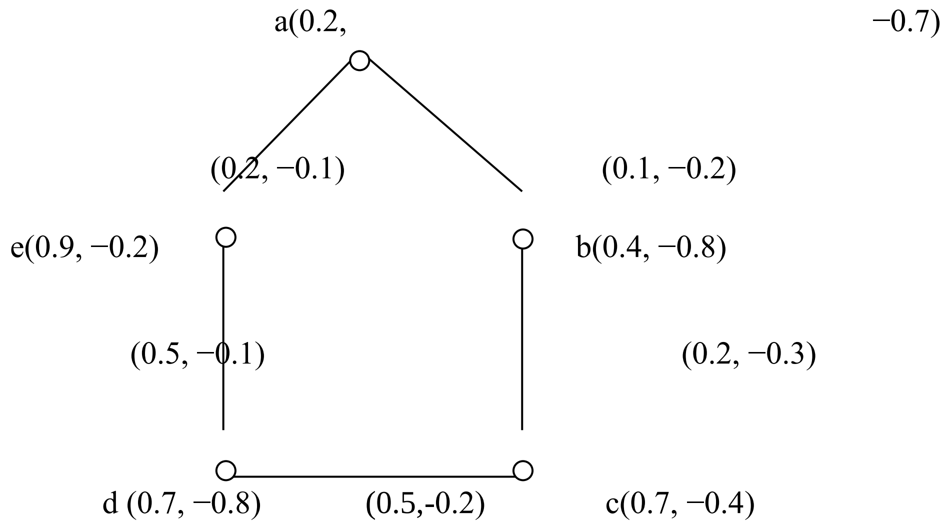


Figure.1

$d(a) = (0.3, -0.3)$, $d(b) = (0.3, -0.5)$, $d(c) = (0.7, -0.5)$, $d(d) = (1, -0.3)$, $d(e) = (0.7, -0.2)$.

Every pair of vertices which are at a distance 2 away from each other have distinct degrees. So this graph is 2^{nd} -neighbourly irregular bipolar fuzzy graph.

Definition 3.3. Let G be a bipolar fuzzy graph on $G^*(V, E)$. Then G is said to be m^{th} - neighbourly totally irregular bipolar fuzzy graph, if every pair of vertices which are at a distance m away from each other have distinct total degrees.

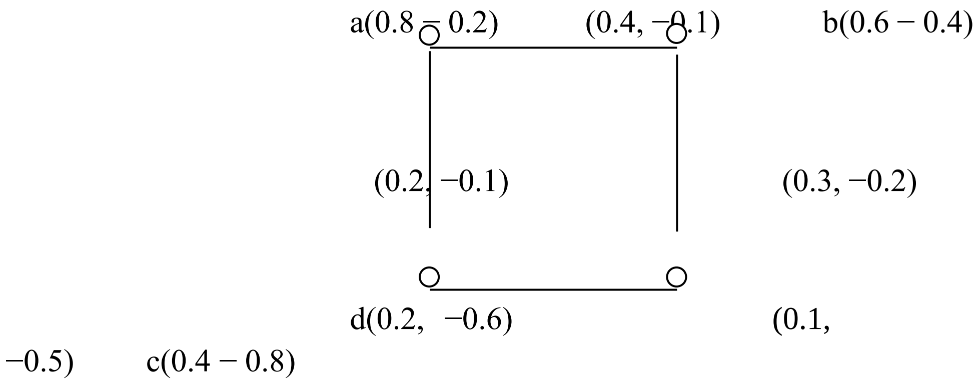
Example 3.4. In the above fig 1,

$td(a) = (0.5, -1)$, $td(b) = (0.7, -1.3)$, $td(c) = (1.4, -0.9)$, $td(d) = (1.7, -1.1)$, $td(e) = (1.6, -0.4)$.

Every pair of vertices which are at a distance 2 away from each other have distinct total degrees. So this graph is 2^{nd} -neighbourly totally irregular bipolar fuzzy graph .

Remark 3.5. A m^{th} - neighbourly irregular bipolar fuzzy graph which is also a m^{th} -neighbourly totally irregular bipolar fuzzy graph.

Example 3.6. Consider a bipolar fuzzy graph on G*(V, E).



F igure.2

d(a) = (0.6, −0.2), d(b) = (0.7, −0.3), d(c) = (0.4, −0.7), d(d) = (0.3, −0.6).

td(a) = (1.4, −0.4), td(b) = (1.3, −0.7), td(c) = (0.8, −1.5), td(d) = (0.5, −1.2).

Every pair of vertices which are at a distance 2 away from each other having distinct degree and distinct total degree. Hence G is 2^{nd} - neighbourly irregular bipolar fuzzy graph and also 2^{nd} - neighbourly totally irregular bipolar fuzzy graph.

Remark 3.7. A m^{th} - neighbourly totally irregular bipolar fuzzy graph need not be m^{th} - neighbourly irregular bipolar fuzzy graph.

Example 3.8. Consider a bipolar fuzzy graph on G*(V, E).

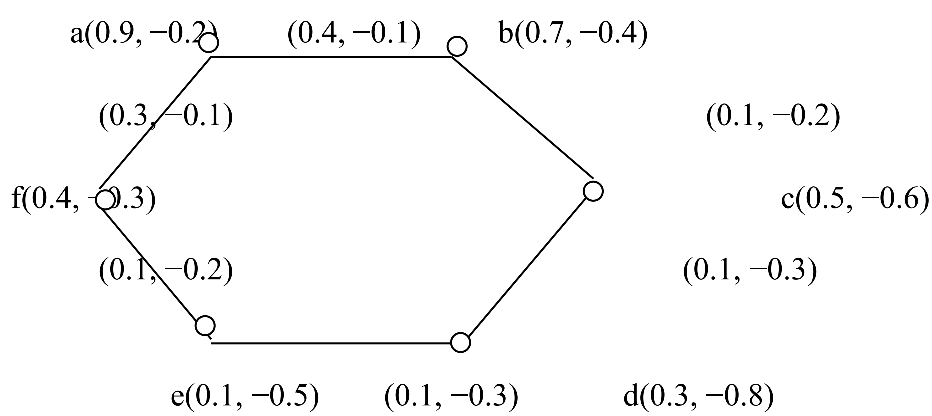


Figure.3

d(a) = (0.7, −0.2), d(b) = (0.5, −0.3), d(c) = (0.2, −0.5), d(d) = (0.2, −0.6), d(e) = (0.2, −0.5), d(f) = (0.4, −0.3). td(a) = (1.6, −0.4), td(b) = (1.2, −0.7), td(c) = (0.7, −1.1), td(d) = (0.5, −1.4), td(e) = (0.3, −1), td(f) = (0.8, −0.6).

Every pair of vertices which are at a distance 2 away from each other have distinct total degree . Hence G is 2^{nd} - neighbourly totally irregular fuzzy graph . But the vertices c and e are at a distance 2 away from each other having same degree . Hence G is not 2^{nd} - neighbourly irregular fuzzy graph.

Remark 3.9. A m^{th} - neighbourly irregular bipolar fuzzy graph need not be m^{th} - neighbourly totally irregular bipolar fuzzy graph.

Example 3.10. Consider a bipolar fuzzy graph on $G^*(V, E)$.

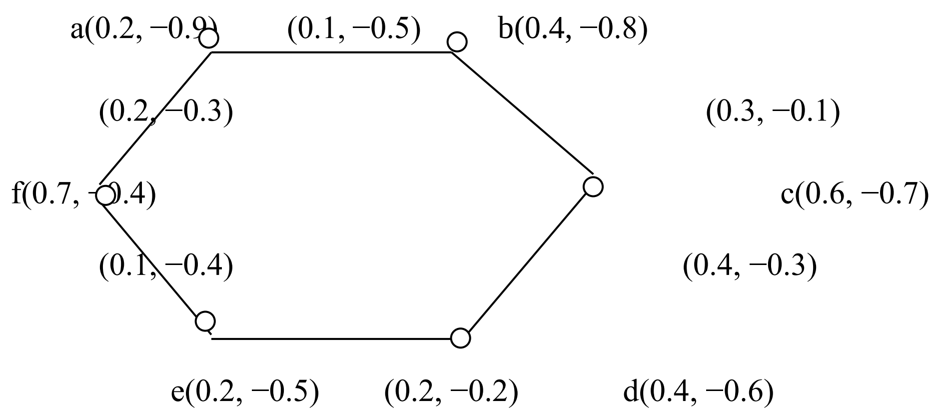


Figure.4

$d(a) = (0.3, -0.8)$, $d(b) = (0.4, -0.6)$, $d(c) = (0.7, -0.4)$, $d(d) = (0.6, -0.5)$, $d(e) = (0.3, -0.6)$, $d(f) = (0.3, -0.7)$. $td(a) = (0.5, -1.7)$, $td(b) = (0.8, -1.4)$, $td(c) = (1.3, -1.1)$, $td(d) = (1, -1.1)$, $td(e) = (0.5, -1.1)$, $td(f) = (1, -1.1)$.

Every pair of vertices which are at a distance two away from each other have distinct degrees . Hence G is 2^{nd} - neighbourly irregular bipolar fuzzy graph. But the vertices d and f are at a distance 2 away from each other have same total degree . Hence G is not 2^{nd} - neighbourly totally irregular fuzzy graph.

Theorem 3.11. Let G : (A, B) be a bipolar fuzzy graph on $G^*(V, E)$. If G is m^{th} - neighbourly irregular bipolar fuzzy graph and A is a constant function. Then G is m^{th} - neighbourly totally irregular bipolar fuzzy graph.

Proof. Let G : (A, B) be a m^{th} - neighbourly irregular bipolar fuzzy graph. Then every pair of vertices which are at a distance m away from each other have distinct degrees. Let (v, w) and (w, x) be the pair of vertices which are at a distance m away from each other such that $d(v) = (k_1, k_2)$, $d(w) = (k_3, k_4)$ and $d(x) = (k_5, k_6)$ where $(k_1, k_2) \neq (k_3, k_4)$, $(k_3, k_4) \neq (k_5, k_6)$. Also, assume that $A(u) = (c_1, c_2)$, for all $u \in V$. Suppose $td(v) = td(w) \Rightarrow d(v) + A(v) = d(w) + A(w) \Rightarrow ((k_1, k_2) + (c_1, c_2)) = ((k_3, k_4) + (c_1, c_2)) \Rightarrow (k_1, k_2) = (k_3, k_4)$ which is a contradiction. So $td(v) \neq td(w)$. Similarly $td(w) \neq td(x)$. Hence the vertices (v, w) and (w, x) have distinct total degrees provided A is a constant function. This is true for every pair of vertices which are at a distance m away from each other in G. Hence G is m^{th} -neighbourly totally irregular bipolar fuzzy graph.

Example 3.12. Consider a bipolar fuzzy graph on $G^*(V, E)$.

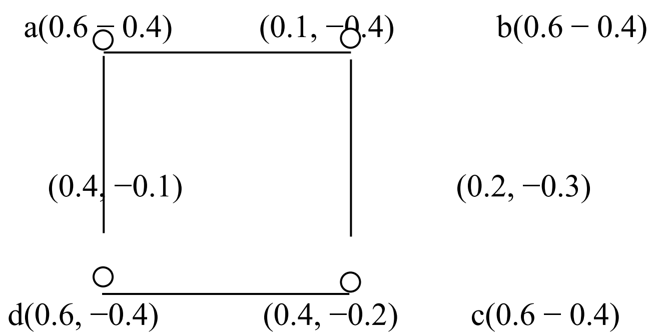


Figure.5

$d(a) = (0.5, -0.5)$, $d(b) = (0.3, -0.7)$, $d(c) = (0.6, -0.5)$, $d(d) = (0.8, -0.3)$.
$td(a) = (1.1, -0.9)$, $td(b) = (1.9, -1.1)$, $td(c) = (1.2, -0.9)$, $td(d) = (1.3, -0.7)$.

Every pair of vertices which are at a distance 2 away from each other have distinct degrees. Hence G is 2^{nd} - neighbourly irregular bipolar fuzzy graph and A is constant function. Hence G is m^{th} - neighbourly totally irregular bipolar fuzzy graph.

Theorem 3.13. Let G : (A, B) be a bipolar fuzzy graph on $G^*(V, E)$. If G is m^{th} -neighbourly totally irregular bipolar fuzzy graph and A is a constant function, then G is m^{th} -neighbourly irregular bipolar fuzzy graph.

Proof. Let G : (A, B) be a m^{th} - neighbourly totally irregular bipolar fuzzy graph. Then every pair of vertices which are at a distance m away from each other have distinct total degrees. Let v, w be the vertices which are at a distance m away from each other and w, x be the vertices which are at a distance m away from each other, such that $td(v) = (k_1, k_2)$, $td(w) = (k_3, k_4)$ and $td(x) = (k_5, k_6)$ where $(k_1, k_2) \neq (k_3, k_4)$,$(k_3, k_4) \neq (k_5, k_6)$. Also assume that $A(u) = (c_1,c_2)$, for all $u \in V$. Suppose $td(v) \neq td(w) \Rightarrow d(v) + A(v) \neq d(w) + A(w) \Rightarrow d(v) - d(w) \neq A(w) - A(v) \neq 0 \Rightarrow d(v) \neq d(w)$. Similarly $d(w) \neq d(x)$. So, G is m^{th} - neighbourly irregular bipolar fuzzy graph.

Remark 3.14. The above two theorems jointly yield the following result. Let G be a bipolar fuzzy graph on $G^*(V, E)$. If A is a constant function then G is m^{th} - neighbourly irregular bipolar fuzzy graph, if and only if G is m^{th} - neighbourly totally irregular bipolar fuzzy graph.

Remark 3.15. Let G : (A, B) be a bipolar fuzzy graph on $G^*(V, E)$. If G is both m^{th} - neighbourly irregular bipolar fuzzy graph and m^{th} - neighbourly totally irregular bipolar fuzzy graph , then A need not be a constant function.

Example 3.16. Consider a bipolar fuzzy graph on $G^*(V, E)$.

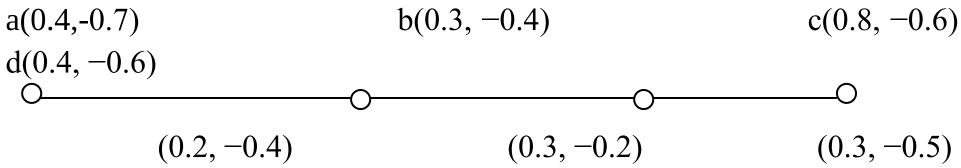


Figure.6

$$d(a) = (0.2, -0.4),\ d(b) = (0.5, -0.6),\ d(c) = (0.6, -0.7),\ d(d) = (0.3, -0.5)$$

$$td(a) = (0.6, -1.1),\ td(b) = (0.8, -1),\ td(c) = (1.4, -1.3),\ td(d) = (0.7, -1.1).$$

Hence G is both 2^{nd} - neighbourly irregular bipolar fuzzy graph and 2^{nd} - neighbourly totally irregular bipolar fuzzy graph, but A is not a constant function.

Theorem 3.17. Let G : (A, B) be a bipolar fuzzy graph on $G^*(V, E)$. If G is strongly irregular bipolar fuzzy graph, then G is m^{th} - neighbourly irregular bipolar fuzzy graph.

Proof. Let G : (A, B) be a bipolar fuzzy graph on $G^*(V, E)$. Assume that G is strongly irregular bipolar fuzzy graph. Then every pair of vertices in G have distinct degrees ⇒ all the vertices in G has distinct degree ⇒ every pair of vertices which are at a distance m away from each other have distinct degree in G. Hence G is m^{th} - neighbourly irregular bipolar fuzzy graph.

Theorem 3.18. Let G : (A, B) be a bipolar fuzzy graph on $G^*(V, E)$. If G is strongly totally irregular bipolar fuzzy graph then G is m^{th} - neighbourly irregular bipolar fuzzy graph.

Proof. Proof is similar to above theorem 3.17.

Remark 3.19. Converse of the theorem 3.17 and 3.18 need not be true.

Example 3.20. Consider a bipolar fuzzy graph on $G^*(V, E)$.

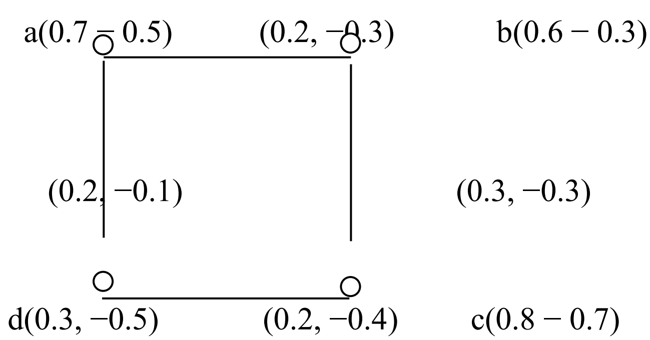


Figure.7

$d(a) = (0.4, -0.4)$, $d(b) = (0.5, -0.6)$, $d(c) = (0.5, -0.7)$, $d(d) = (0.4, -0.5)$
$td(a) = (1.1, -0.9)$, $td(b) = (1.1, -0.9)$, $td(c) = (1.3, -1.4)$, $td(d) = (0.7, -1)$.

So every pair of vertices which are at a distance 2 away from each other have distinct degrees. Hence G is 2^{nd} - neighbourly irregular bipolar fuzzy graph. But the vertices b and c have same degrees. Hence G is not strongly irregular bipolar fuzzy graph.
Also every pair of vertices which are at a distance 2 away from each other have distinct total degrees. Hence G is 2^{nd} - neighbourly irregular bipolar fuzzy graph. But the vertices a and b have the same total degrees. Hence G is not strongly totally irregular bipolar fuzzy graph.

4 m^{th} -Neighbourly irregularity on a cycle with some specific membership function

Theorem 4.1. Let G : (A, B) be a bipolar fuzzy graph on $G^*(V, E)$ an even cycle of length n . If A takes distinct membership values and B is constant function or alternate edges takes same membership values, then G is m^{th} - neighbourly totally irregular bipolar fuzzy graph.

Proof. Case 1: Suppose B is a constant $d(u) = (k_1, k_2)$, for all $u \in V$. We have $td(u) = d(u) + A(u)$. Since A takes distinct membership values, every pair of vertices which are at a distance m away from each other have distinct total degrees. Hence G is m^{th} - neighbourly totally irregular bipolar fuzzy graph.

Case 2: Alternate edges takes same membership values. Let w and y be the vertices are at a distance m away from each other. Since A takes distinct membership values, let $A(w) = (c_1, c_2)$ and $A(y) = (c_3, c_4)$. Now $td(w) = (c_1, c_2) + (k_1, k_2) + (k_3, k_4)$ and $td(y) = (c_3, c_4) + (k_1, k_2) + (k_3, k_4) \Rightarrow td(w) \neq td(y)$. So the vertices w and y have distinct total degrees. This is true for every pair of vertices which are at a distance m away from each other in G. Hence G is m^{th} - neighbourly totally irregular bipolar fuzzy graph.

Theorem 4.2. Let G : (A, B) be a bipolar fuzzy graph on $G^*(V, E)$ a cycle of length n. If the positive membership function of the edges takes the value $c_1, c_2, \ldots, c_n$ and the negative membership function of the edges takes the value $d_1, d_2, \ldots, d_n$ such that $c_1 < c_2 < \ldots < c_n$ and $d_1 > d_2 > \ldots > d_n$, then G is m^{th} - neighbourly irregular bipolar fuzzy graph.

Proof. Let $e_1, e_2, \ldots, e_n$ be the edges of the Cycle C_n taking membership values $(c_1, d_1),(c_2, d_2), \ldots,(c_n, d_n)$.
Then $d(v_1) = (c_1, d_1) + (c_n, d_n) = ((c_1 + c_n),(d_1 + d_n))$.
For $i = 2, 3, \ldots, n - 1$,
we have $d(v_i) = (c_{i-1}, d_{i-1}) + (c_{i+1}, d_{i+1}) = (c_{i-1} + c_{i+1}, d_{i-1} + d_{i+1})$.
$d(v_n) = (c_1 + c_{n-1}, d_1 + d_{n-1})$.

Here all the vertices have distinct degrees . So every pair of vertices which are at a distance m away from each other have distinct degrees. Hence G is m^{th} - neighbourly irregular bipolar fuzzy graph.

Remark 4.3. Even if the positive membership function of the edges takes the value $c_1, c_2, \ldots, c_n$ and the negative membership function of the edges takes the value $d_1, d_2, \ldots, d_n$ such that $c_1<c_2< \ldots <c_n$ and $d_1>d_2> \ldots >d_n$, then G is not m^{th} - neighbourly totally irregular bipolar fuzzy graph.

Example 4.4. Consider a bipolar fuzzy graph on $G^*(V, E)$.

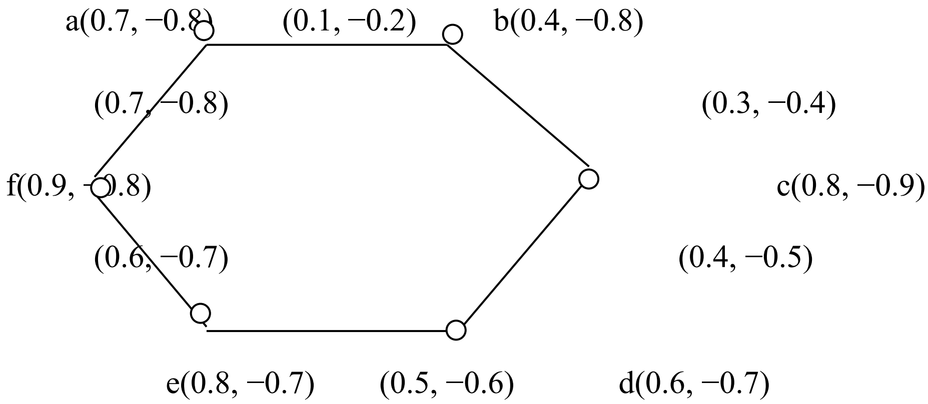


Figure.8

$d(a) = (0.8, -1)$, $d(b) = (0.4, -0.6)$, $d(c) = (0.7, -0.9)$, $d(d) = (0.9, -1.1)$, $d(e) = (1.1, -1.3)$, $d(f) = (1.3, -1.5)$. $td(a) = (1.5, -1.8)$, $td(b) = (0.8, -1.4)$, $td(c) = (1.5, -1.8)$,

$td(d) = (1.5, -1.8)$, $td(e) = (1.9, -2.0)$, $td(f) = (2.2, -2.3)$.

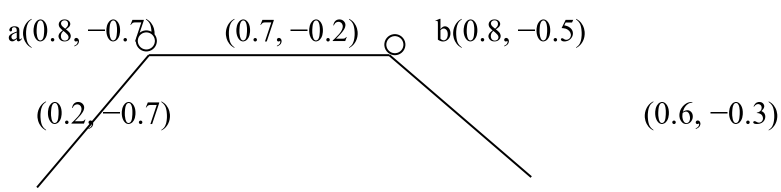
Here the vertices a and c which are at a distance 2 away from each other having same total degrees. Hence G is not 2^{nd} -neighbourly totally irregular bipolar fuzzy graph.

Theorem 4.5. Let G : (A, B) be a bipolar fuzzy graph on $G^*(V, E)$ a cycle length n. If the positive membership function of the edges takes the value $c_1, c_2, \ldots, c_n$ and the negative membership function of the edges takes the value $d_1, d_2, \ldots, d_n$ such that $c_1>c_2> \ldots >c_n$ and $d_1<d_2< \ldots <d_n$, then G is m^{th} - neighbourly irregular bipolar fuzzy graph.

Proof. Proof is similar to above theorem 4.2.

Remark 4.6. Even if the positive membership function of the edges takes the value $c_1, c_2, \ldots, c_n$ and the negative membership function of the edges takes the value $d_1, d_2, \ldots, d_n$ such that $c_1>c_2> \ldots >c_n$ and $d_1<d_2< \ldots <d_n$, then G is not m^{th} - neighbourly totally irregular bipolar fuzzy graph.

Example 4.7. Consider a bipolar fuzzy graph on $G^*(V, E)$.



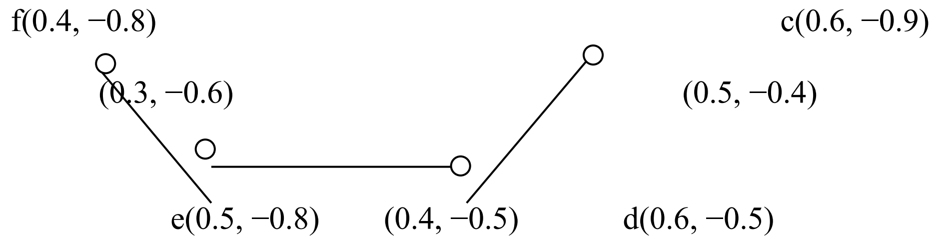


Figure.9

d(a) = (0.9, −0.9), d(b) = (1.3, −0.5), d(c) = (1.1, −0.7), d(d) = (0.9, −0.9), d(e) = (0.7, −1.1), d(f) = (0.5, −1.3). td(a) = (1.7, −1.6), td(b) = (2.1, −1.0), td(c) = (1.7, −1.6),
td(d) = (1.5, −1.4), td(e) = (1.2, −1.9), td(f) = (0.9, −2.1) .

Here the vertices a and c which are at a distance 2 away from each other have same total degrees. Hence G is not 2^{nd} -neighbourly totally irregular bipolar fuzzy graph.

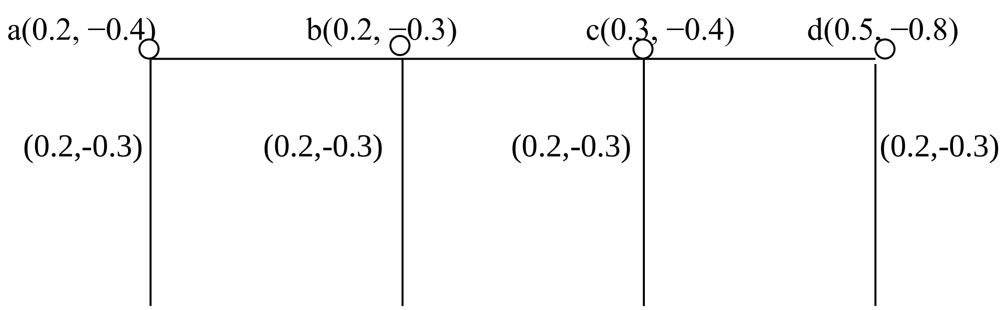
5 m^{th} - Neighbourly irregularity on a ladder with some specific membership functions

Theorem 5.1. If the ladder on n vertices takes membership values $(c_1, d_1),(c_2, d_2), \ldots, (c_n, d_n)$ are all either odd or even,such that $c_1<c_2< \ldots <c_n$ and $d_1>d_2> \ldots >d_n$ and middle edges takes same membership values even or odd (k_1, k_2) . Then G is m^{th} - neighbourly irregular bipolar fuzzy graph.

Proof. Let u_i be the set of vertices in P_n.
$d(u_1) = (c_1, d_1) + (k_1, k_2)$.
$d(u_i) = (c_{i-1}, d_{i-1}) + (c_{i+1}, d_{i+1}) + (k_1, k_2)$,
for $i = 2, 3, \ldots, n - 1$ and
$d(u_n) = (c_{n-1}, d_{n-1}) + (k_1, k_2) = ((c_{n-1} + k_1),(d_{n-1} + k_2))$.
Every vertex which are at a distance m away from each other have distinct degrees. Hence G is m^{th} - neighbourly irregular bipolar fuzzy graph.

Remark 5.2. Even if the ladder on n vertices takes membership values $(c_1, d_1),(c_2, d_2), \ldots,(c_n, d_n)$ are all either odd or even , such that $c_1<c_2< \ldots <c_n$ and $d_1>d_2> \ldots >d_n$ and middle edges takes same membership values even or odd (k_1, k_2), then G need not be m^{th} - neighbourly totally irregular bipolar fuzzy graph.

Example 5.3. Consider a bipolar fuzzy graph on $G^*(V, E)$.



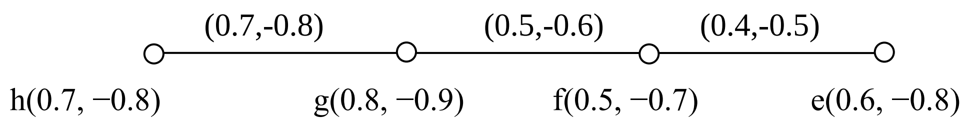


Figure.10

td(a) = (0.5, −0.9), td(b) = (1.0, −1.2), td(c) = (1.2, −1.6), td(d) = (1.0, −1.5), td(e) = (1.2, −1.6), td(f) = (1.6, −2.1), td(g) = (2.2, −2.6), td(h) = (1.6, −1.9).

Here the vertices c and e which are at a distance 2 away from each other have same total degrees. Hence G is not 2^{nd} - neighbourly totally irregular bipolar fuzzy graph.

Theorem 5.4. Even if the ladder on n vertices takes membership values $(c_1, d_1),(c_2, d_2), . . . ,(c_n, d_n)$ are all either odd or even , such that $c_1>c_2> . . . >c_n$ and $d_1<d_2< . . . <d_n$ and middle edges takes same membership values even or odd (k_1, k_2), then G is m^{th} - neighbourly irregular bipolar fuzzy graph.

Proof. Proof is similar to above theorem 5.1.

Remark 5.5. If the ladder on n vertices takes membership values $(c_1, d_1),(c_2, d_2), . . . ,(c_n, d_n)$ are all either odd or even , such that $c_1>c_2> . . . >c_n$ and $d_1<d_2< . . . <d_n$ and middle edges takes same membership values even or odd (k_1, k_2), then G need not be m^{th} - neighbourly totally irregular bipolar fuzzy graph.

Example 5.6. Consider a bipolar fuzzy graph on $G^*(V, E)$.

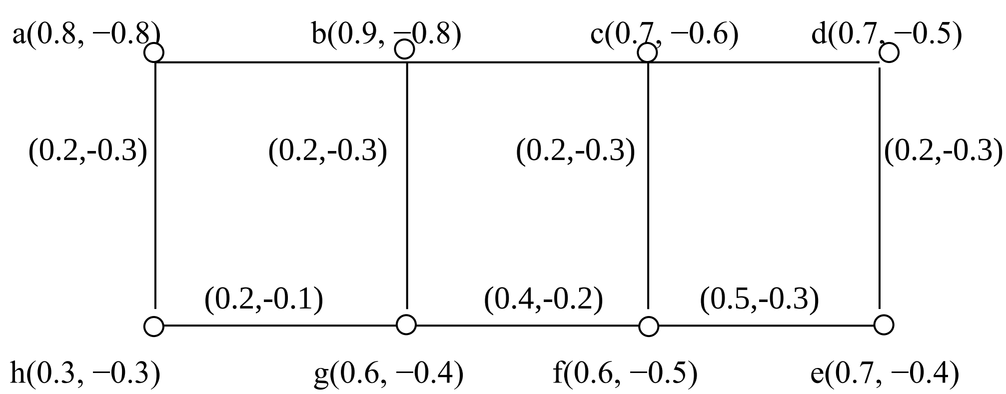


Figure.11

td(a) = (1.8, −1.8), td(b) = (2.6, −2.4), td(c) = (2.2, −2), td(d) = (1.5, −1.3), td(e) = (1.4, −1.0), td(f) = (1.7, −1.3), td(g) = (1.4, −1), td(h) = (0.7, −0.7).

Here the vertices e and g which are at a distance 2 away from each other have same total degrees. Hence G is not 2^{nd} - neighbourly totally irregular bipolar fuzzy graph.

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