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Edge Trimagic Graceful Labeling Of Some Star Graphs

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ABSTRACT

A (p, q) graph G is called an edge trimagic graceful if there exists a bijection f : V(G) ∪ E(G) → {1, 2, 3, …, p + q} such that for each edge xy in E(G), |f (x) – f (xy) + f (y)| = C_1 or C_2 or C_3, where C_1, C_2 and C_3 are constants. In this paper, we proved that the bistar, double star, triple star graph are edge trimagic graceful graphs.

Key words: Graph, trimagic, graceful, trimagic graceful, bistar, double star, triple star.

AMS Subject Classification: 05C78

1. INTRODUCTION

Labeling of a graph G is an assignment of labels to either the vertices or the edges or both subject to certain conditions. Graph labeling was first introduced in1960's. Graph labeling are of many types such as magic, bimagic, trimagic, antimagic, graceful, harmonious, equitable, etc. In this paper, we are going to study about trimagic graceful labeling of some ladder family graphs.

Magic labeling was introduced by Sedlacek [1]. In 1970, Kotzig and Rosa [2] defined, an edge magic labeling of graph G is a bijection f: V∪E → {1, 2, …, p + q} such that, for each edge uv ∈ E(G), f(u) + f(uv) + f(v) is a magic constant. In 2004, Edge Bimagic labeling was introduced by J. B. Babujee [6] as a graph G with a bijection f: V ∪ E → {1, 2, …, p + q} such that, for each edge uv∈ E(G), f(u) + f(uv) + f(v) is either k_1 or k_2.

In 2013, C. Jayasekaran, M. Regees and C. Davidraj [6] introduced the edge trimagic total labeling of graphs. An edge trimagic total labeling of a (p, q) graph G is a bijective function f: $V(G) \cup E(G) \rightarrow \{1, 2, \ldots, p + q\}$ such that for each edge $xy \in E(G)$, the value of f(x) + f(xy) + f(y) is equal to any of the distinct constants k_1 or k_2 or k_3. A (p, q) graph G is called an edge magic graceful if there exists a bijection f: $V(G) \cup E(G) \rightarrow \{1, 2, \ldots, p + q\}$ such that for each edge xy in E(G) the value of $|f(x) + f(y) - f(xy)| = k$, a constant [3]. The graph G is said to be super edge magic graceful if $V(G) = \{1, 2, \ldots, p\}$.

A star graph S_k is a complete bipartite graph $K_{1,n}$ of order n + 1 is a tree on n + 1 vertices, in a star graph one vertex has degree n and the remaining n vertices have degree 1. Stars may also be described as the only connected graphs in which at most one vertex has degree greater than one. $B_{m,n}$ is a (m, n) Bistar obtained from two disjoint copies of $K_{1,m}$ by joining the central vertices by an edge. The Double Star $K_{1,n,n}$ is a tree obtained from the star $K_{1,n}$ by adding a new pendent edge to each of the existing n pendent vertices. The Triple Star $K_{1,n,n,n}$ is a tree obtained from the double star $K_{1,n,n}$ by adding a new pendent edge to each of the existing n pendent vertices [7]. We use the Dynamic Survey of Graph Labeling by Joseph A. Gallian [4] for more references. In this paper, we have proved bistar, double star, triple star, splitting graph of a star graph are edge trimagic graceful graphs.

2. MAIN RESULTS

Theorem 2.1: The Bistar $B_{m,n}$ graph admits an edge trimagic graceful labeling and super edge trimagic graceful labeling for all n and for $m \geq 2$.

Proof: Let the vertex set of the Bistar graph be, $V(B_{m,n}) = \{u, v, u_i, v_j \,/\, 1 \leq i \leq m, 1 \leq j \leq n\}$ and the edge set be $E(B_{m,n}) = \{uv, uw_i, uu_i, vv_j \,/\, 1 \leq i \leq m, 1 \leq j \leq n\}$. Then $B_{m,n}$ has m + n + 2 vertices and m + n + 1 edges.
Define a bijection $\varphi: V \cup E \rightarrow \{1, 2, 3, ..., 2m + 3n + 2\}$ such that
$\varphi(u) = 2m + n + 1$; $\varphi(v) = 2m + n + 2$
$\varphi(u_i) = i, 1 \leq i \leq m$; $\varphi(v_j) = m + j, 1 \leq j \leq n$
$\varphi(uv) = 2m + n + 3$; $\varphi(uu_i) = m + n + i , 1 \leq i \leq m$
and $\varphi(vv_j) = 2m + n + j + 3, 1 \leq i \leq n$
For each edge $uv \in E(B_{m,n})$, $|\varphi(u) - \varphi(uv) + \varphi(v)|$ will get any one of the constants $c_1 = |2m + n|$, $c_2 = |m + 1|$ and $c_3 = |m - 1|$. Therefore the Bistar graph $B_{m,n}$ admits an edge trimagic graceful labeling for all n and $m \geq 2$. Since the Bistar graph $B_{m,n}$ has m + n + 2 vertices and these m + n + 2 vertices have labels 1, 2, ..., m + n + 2 for all n and $m \geq 2$, hence the graph $B_{m,n}$ is a super edge trimagic graceful.
Example 2.2: An edge trimagic graceful labeling of $B_{5,4}$ is given in figure 2.1.

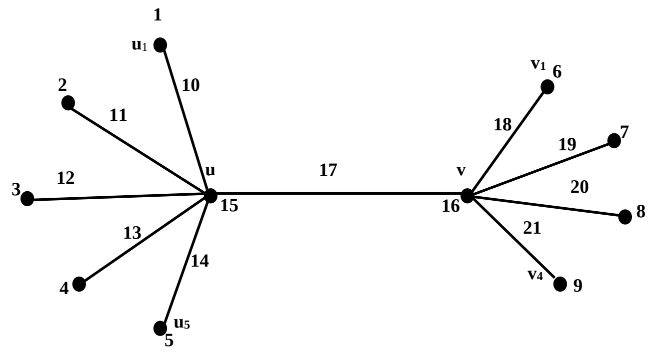

Figure 2.1: $B_{5,4}$ with $c_1 = 14$, $c_2 = 6$ and $c_3 = 4$.

Theorem 2.3: The Double Star graph $K_{1,n,n}$ admits an edge trimagic graceful labeling and super edge trimagic graceful labeling for $n > 2$.

Proof: The vertex set of the Double Star graph be, V ($K_{1,n,n}$) = { $v_0, v_1, v_2, \ldots, v_{2n}$ }and the edge set be E ($K_{1,n,n}$) ={v_0v_i, v_iv_{i+1} / $1 \le i \le n$ }. Then the Double Star graph $K_{1,n,n}$ have 2n + 1 vertices and 2n edges.

Case 1: n is odd

Define a bijection φ: V ∪ E → {1, 2, 3, ..., 4n + 1} such that

$\varphi(v_0) = 1$; $\varphi(v_i) = i + 1$, $1 \le i \le n$

$$\varphi(v_{n+i}) = \begin{cases} n + i + 1, & 1 \le i \le \frac{n+1}{2} \\ n + i + 1, & \frac{n+3}{2} \le i \le n \end{cases}$$

$$\varphi(v_i v_{n+i}) = \begin{cases} 3n + 2i, & 1 \le i \le \frac{n+1}{2} \\ 2n + 2i, & \frac{n+3}{2} \le i \le n \end{cases}$$

and $\varphi(v_0v_i) = 2n + i + 1$, $1 \le i \le n$

For each edge uv ∈ E ($K_{1,n,n}$), $|\varphi(u) - \varphi(uv) + \varphi(v)|$ will get any one of the constants $c_1 = |1 - 2n|$, $c_2 = |2 - 2n|$ and $c_3 = |2 - n|$. Therefore the Double star graph $K_{1,n,n}$ admits an edge trimagic graceful labeling for odd $n > 2$.

Case 2: n is even

Define a bijection φ: V ∪ E → {1, 2, 3, ..., 4n+1} such that

$\varphi(v_0) = 1$; $\varphi(v_i) = i + 1$, $1 \le i \le n$

$$\varphi(v_{n+i}) = \begin{cases} n + i + 1, & 1 \le i \le \frac{n}{2} \\ n + i + 1, & \frac{n+4}{2} \le i \le n \end{cases}$$

$$\varphi(v_i v_{n+i}) = \begin{cases} 3n + 2i, & 1 \le i \le \frac{n}{2} \\ 2n + 2i + 1, & \frac{n+4}{2} \le i \le n \end{cases}$$

and $\varphi(v_0v_i) = 2n + i + 1$, $1 \le i \le n$

Hence for each edge uv ∈ E ($K_{1,n,n}$), │ φ(u) – φ(uv) + φ(v) │ will get any one of the constants c_1 =│1 – 2n │, c_2 =│2 – 2n │and c_3 = │1 – n│. Therefore the Double star graph $K_{1,n,n}$ admits an edge trimagic graceful labeling for even n > 2.
Since the Double star graph $K_{1,n,n}$ has 2n + 1 vertices and these 2n + 1 vertices have labels 1, 2, ..., 2n+ 1 for all n > 2, hence $K_{1,n,n}$ is a super edge trimagic graceful.
Example 2.4: An edge trimagic graceful labeling of $K_{1,6,6}$ is given in figure 2.2.

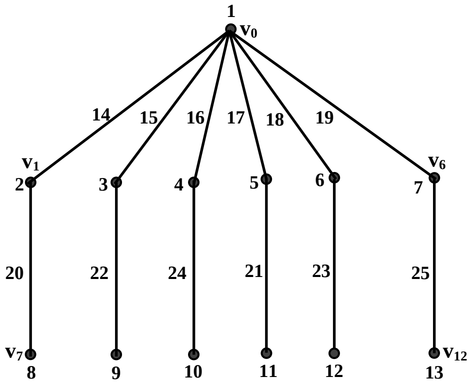

Figure 2.2: $K_{1,6,6}$ with c_1 = 11, c_2 = 10 and c_3 = 5.

Theorem 2.5: The Triple Star graph $K_{1,n,n,n}$ admits an edge trimagic graceful labeling and super edge trimagic graceful labeling for n ≥ 2.
Proof: Let the vertex set of 2he Triple Star graph be, V ($K_{1,n,n,n}$) = { v_0, v_1, v_2, …, v_{3n} } and the edge set be E ($K_{1,n,n,n}$) = { v_0v_i, v_iv_{n+i}, $v_{n+i}v_{2n+i}$ / 1 ≤ i ≤ n }. Then the triple star graph $K_{1,n,n,n}$ has 3n + 1 vertices and 3n edges.
Define a bijection φ: V ∪ E → {1, 2, 3, ..., 6n + 1} such that
φ(v_0) = 1 ; φ(v_i) = i + 1, 1 ≤ i ≤ n
φ(v_{n+i}) = n + i + 1, 1 ≤ i ≤ n; φ(v_{2n+i}) = 2n + i + 1, 1 ≤ i ≤ n
φ(v_0v_i) = 3n + i + 1, 1 ≤ i ≤ n; φ(v_iv_{n+i}) = 4n + 2i, 1 ≤ i ≤ n
and φ($v_{n+i}v_{2n+i}$) = 4n + 2i + 1, 1 ≤ i ≤ n
For each edge uv ∈ E ($K_{1,n,n,n}$), │ φ(u) – φ(uv) + φ(v) │ will get any one of the constants c_1 =│1 – 3n │, c_2 =│2 – 3n │and c_3 = │1 – n│. Therefore the Triple star graph $K_{1,n,n,n}$ admits an edge trimagic graceful labeling for n ≥ 2. Since the Triple star graph $K_{1,n,n,n}$ has 2n + 1 vertices and these 2n + 1 vertices have labels 1, 2, ..., 2n+ 1 for all n ≥ 2, hence the graph $K_{1,n,n,n}$ is a super edge trimagic graceful.

Example 2.6: An edge trimagic graceful labeling of $K_{1,5,5,5}$ is given in figure 2.3.

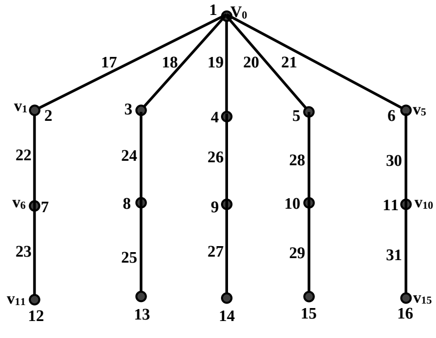

Figure 2.3: $K_{1,5,5,5}$ with c_1 = 14, c_2 = 13 and c_3 = 4.

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